



2.1.7 Two-dimensional vectors versus complex numbers





Introduction

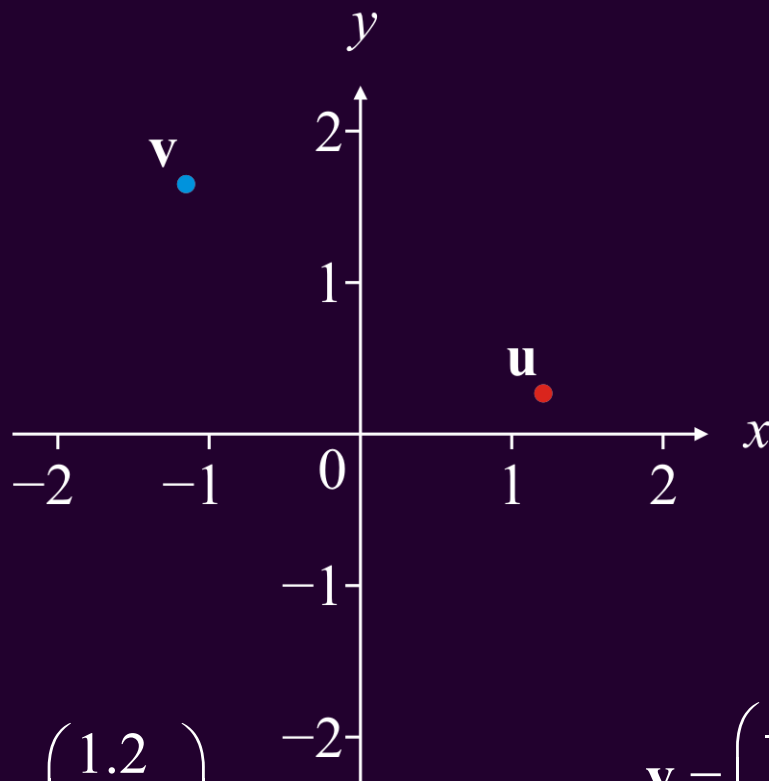
- In this topic, we will
 - Observe the superficial relationship between 2-d vectors and complex numbers
 - We will discuss the differences





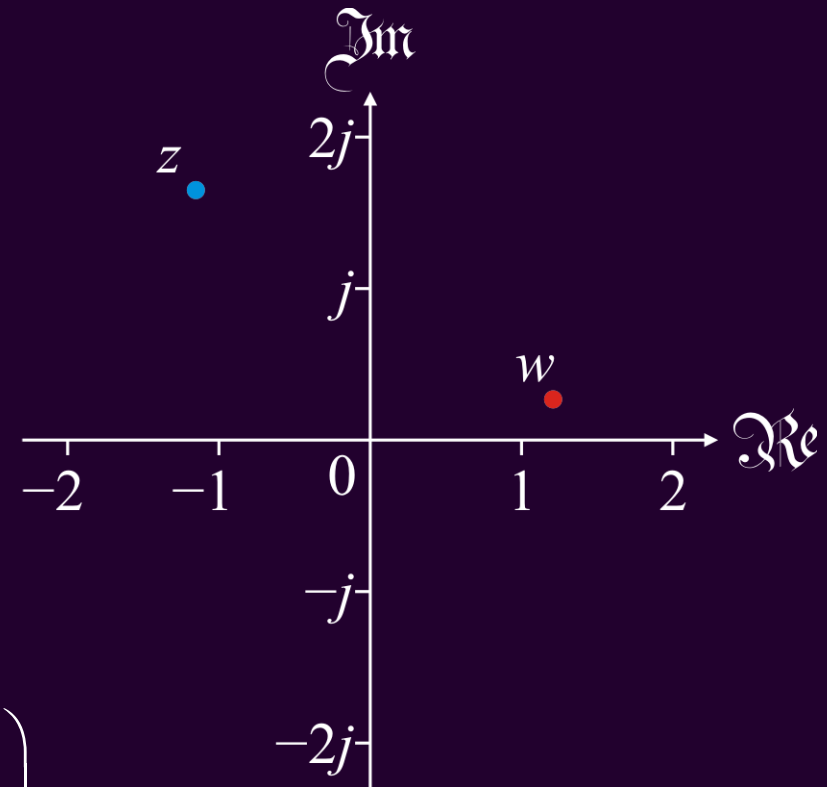
2-d vectors and complex numbers

- We have described 2-dimensional vectors and complex numbers



$$\mathbf{u} = \begin{pmatrix} 1.2 \\ 0.27 \end{pmatrix}$$

$$w = 1.2 + 0.27j$$



$$\mathbf{v} = \begin{pmatrix} -1.15 \\ 1.65 \end{pmatrix}$$

$$z = -1.15 + 1.65j$$





2-d vectors and complex numbers

- For a vector,
each entry represents a separate piece of information
- A complex number is a single value,
no different than a real number
- For a vector,
we will not be able to have a useful multiplication that
works in $\mathbf{R}^n$
- Multiplication of complex numbers is well defined and has
significance that is applicable





2-d vectors and complex numbers

- For a vector,
there is no significance to the vector $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- For complex numbers,
the value 1 is that value such that $1z = z$ for all z and
every non-zero z has a reciprocal such that $zz^{-1} = 1$

- Recall that $\mathbf{u} = \begin{pmatrix} 7.6 \\ -8.2 \end{pmatrix}$ represents a point on the plane,

yet, there is no significant relationship between this value and

$$\mathbf{v} = \begin{pmatrix} 0.0608 \\ 0.0656 \end{pmatrix} \quad (7.6 - 8.2j)(0.0608 + 0.0656j) = 1$$





2-d vectors and complex numbers

- The question is: what are we modeling
 - There are only limited numbers of applications of complex numbers
 - Finding solutions to differential equations
 - Simulating sinusoidal functions with equal periods
 - Specifically, alternating current
 - Quantum mechanics
 - 2-dimensional vectors are more ubiquitous





Historical comment

- Prior to vectors becoming a topic of study in their own right,
 - William Rowan Hamilton introduced *quaternions*, which had four components $\alpha + \beta i + \gamma j + \delta k$
 - Unfortunately, it is not commutative...
 - There is an eight-component number system, the *octonians*, but for them, multiplication is not even associative
- One cannot create such systems where multiplication is reasonably well defined (e.g., existence of reciprocals) for anything other than powers of two...





Summary

- Following this topic, you now
 - Understand that 2-dimensional vectors and complex numbers represent separate ideas
 - Are aware that the first has two components, while the second is a single value
 - Understand that for vectors, there is no significance to the reciprocals to their corresponding complex value





References

- [1] https://en.wikipedia.org/wiki/Complex_number





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

<https://www.rbg.ca/>

for more information.





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